التكامل

إضافة قوانين للتكامل:

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \arg sh \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \arg ch \frac{x}{a} + c$$

تمرين:

$$I = \int \frac{dx}{x\sqrt{2-\ln^2 x}}$$

$$t = \ln x \quad , \quad dt = \frac{1}{x}\,dx$$

$$I = \int \frac{dt}{\sqrt{2-t^2}} = \arcsin \frac{\ln x}{\sqrt{2}} + c$$

حالات تكامل بالتجزئة:

$$\int x^n e^{ax}\,dx \quad , \quad \int x^n \sin x\,dx$$

$$\int x^n \cos x\,dx$$

$$u = x^n \Rightarrow du = n x^{n-1}\,dx$$

$$dv = \cos x\,dx \Rightarrow v = \sin x$$

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$$I = u\,v - \int v\,du$$

$\int x^n . \ln x . dx$

$\int x^n . \text{arc} . \tan x\, dx$

$\int x^n . \text{arc} . \sin x\, dx$

$\int x^n . \text{arc} . \cos x\, dx$

$\int x^n . \text{arg} . \text{sh} x\, dx$

($x^n \rightarrow dv$; arc / arg part $\rightarrow u$)

$dv = x^n dx$

تمرين :

$$I = \int (\tan^2 x + 1 - 1)\, dx$$

$$\tan x - x + C$$

- $\int \sin 2x . e^{3x} dx$

$u = \sin 2x \Longrightarrow du = 2\cos 2x . dx$

$dv = e^{3x} . dx \Longrightarrow v = \frac{1}{3} e^{3x}$

$$I = u . v - \int v . du$$

$$I = \frac{1}{3} e^{3x} . \sin 2x - \frac{2}{3} \int e^{3x} . \cos 2x\, dx$$

$u = \cos 2x \Longrightarrow du = -2\sin 2x\, dx$

$dv = e^{3x} dx \Longrightarrow v = \frac{1}{3} e^{3x}$

$$I = \frac{1}{3} e^{3x} \sin 2x - \frac{2}{3} \left[\frac{1}{3} e^{3x} \cos 2x + \frac{2}{3} \int e^{3x} . \sin 2x . dx \right]$$

($\int e^{3x} . \sin 2x . dx = I$)

$$I = \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x - \frac{4}{9} I + C$$

$$\frac{4}{9} I + I = \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x + C$$

$$\frac{13}{9} I = \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x + C$$

$$I = \frac{9}{13} \left(\frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x \right) + C$$

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- $\int x^5 \ln x \, dx =$

$u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = x^5 dx \Rightarrow v = \frac{x^6}{6}$

$I = \frac{x^6}{6} \ln x - \frac{1}{6} \int x^5 dx$

- $\int \arcsin x \cdot dx$

$u = \arcsin x \Rightarrow du = \frac{dx}{\sqrt{1-x^2}}$

$dv = dx \Rightarrow v = x$

$I = x \arcsin x - \int \frac{2x}{2\sqrt{1-x^2}} dx$

$I = x \arcsin x + \sqrt{1-x^2} + C$

- $\int \frac{\sin 2x}{\sqrt{3-\cos^4 x}} dx$

$t = \cos^2 x \Rightarrow dt = \underbrace{-2\sin x \cdot \cos x}_{\sin 2x} dx$

$I = -\int \frac{dt}{\sqrt{3-t^2}}$

$= -\arcsin \frac{t}{\sqrt{3}} + C$

$= -\arcsin \frac{\cos^2 x}{\sqrt{3}} + C$

- $\int \frac{x}{9+x^4} dx$

$t = x^2 \Rightarrow dt = 2x \, dx$

$\frac{1}{2} \int \frac{dt}{9+t^2} = \frac{1}{2} \cdot \frac{1}{3} \arctan \frac{t}{3} + C$

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- $\displaystyle\int \frac{dx}{\cos^2 x\,(1+\tan x)^2}$

$t = 1+\tan x \Rightarrow dt = \frac{1}{\cos^2 x}\,dx$

$$\int \frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{1+\tan x} + C$$

- $\displaystyle\int \ln x \, dx$

$u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$dv = dx \Rightarrow v = x$

$$I = x \cdot \ln x - \int x \cdot \frac{1}{x} \cdot dx$$

$$= x \ln x - x + C$$

- $\displaystyle\int (\ln^2 x)\, dx$

$u = \ln^2 x \Rightarrow du = \frac{2}{x} \ln x \cdot dx$

$dv = dx \Rightarrow v = x$

$$I = x \cdot \ln^2 x - 2\int \ln x \cdot dx$$

$$= x \cdot \ln^2 x - 2\left[x \cdot \ln x - x + C_1\right] + C$$

$C = 2C_1$; حيث

- $\displaystyle\int e^x \cdot \sin x \cdot dx$

$u = e^x \Rightarrow du = e^x \cdot dx$

$dv = \sin x\, dx \Rightarrow v = -\cos x$

$$I = -e^x \cos x + \int e^x \cos x \, dx$$

$u = e^x \Rightarrow du = e^x dx$

$dv = \cos x\, dx \Rightarrow v = \sin x$

$$I = -e^x \cos x + e^x \sin x - \underbrace{\int \sin x\, e^x \cdot dx}_{I}$$

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$2I = -e^x \cos x + e^x \sin x + C$

$I = -\frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C$

- $\int \arctan x \, dx$

$u = \arctan x \implies du = \frac{1}{1+x^2} dx$

$dv = dx \implies v = x$

$I = x \arctan x - \int \frac{x}{1+x^2} dx$

$I = x \arctan x - \frac{1}{2} \ln |1+x^2| + C$

- $\int \frac{x \cdot e^x}{(1+x)^2} dx$

$u = x \cdot e^x \implies du (e^x + x \cdot e^x) dx$

$\underbrace{}_{e^x(1+x)}$

$dv = \frac{1}{(1+x)^2} dx \implies v = -\frac{1}{1+x}$

$I = -\frac{x \cdot e^x}{1+x} + \int e^x \cdot dx$

$I = -\frac{x \cdot e^x}{1+x} + e^x + C$

- $\int \cos(\ln x) \, dx$

$u = \cos(\ln x) \implies du = -\sin(\ln x) \frac{1}{x}$

$dv = dx \implies v = x$

$I = x \cos \ln x + \int \sin \ln x \, dx$

$u = \sin \ln x \implies du = \cos (\ln x) \frac{1}{2}$

$dv = dx \implies v = x$

$I = x \cos(\ln x) + x \cdot \sin(\ln x) - \int \cos(\ln x)$

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RBO HAMAK

$$I = \frac{x \cdot \cos(\ln x) + x \sin(\ln x)}{2} + c$$

الوظيفة :

$$\int \frac{e^x}{x} \cdot dx .$$

انتهت المحاضرة

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